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DESIGN, ANALYSIS AND SIMULATION OF MEMORY-BASED CONTROL ALGORITHMS FOR DUAL RANGE MISSILES

AFOSR #F49620-98-0441

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Executive Summary

The primary objective of this project is to design, analyze, and test new autopilot algorithms for trajectory tracking control of dual range missiles. The main contributions of this project can be summarized as follows. First, a low-order memory-based control method was developed. This method is based on observing and generalizing past system responses and control experience. It does not demand detailed information about the system dynamics. Second, the memory-based method is integrated with a partial inverse dynamics control and applied to achieve missile orientation tracking in the presence of actuator dynamics and aerodynamics. It is shown that this method is effective in dealing with system nonlinearities and uncertainties due to varying flight conditions. The method is also extended to position/orientation tracking. Finally, the developed autopilot algorithms were tested via computer simulation.

DESIGN, ANALYSIS AND SIMULATION OF MEMORY-BASED CONTROL ALGORITHMS FOR DUAL RANGE MISSILES

Yong D. Song

1. Introduction

This project is concerned with autopilot design for dual range missiles. Automatic control is essential for reliable and effective operation of missile systems. As conceptually shown in Figure 1, autopilot (Flight Control System) represents an important element in missile guidance and control. The function of autopilot (FCS) is to control the missile in pitch, yaw, and roll motions via adjusting the surface deflections to ensure missile trajectory tracking.

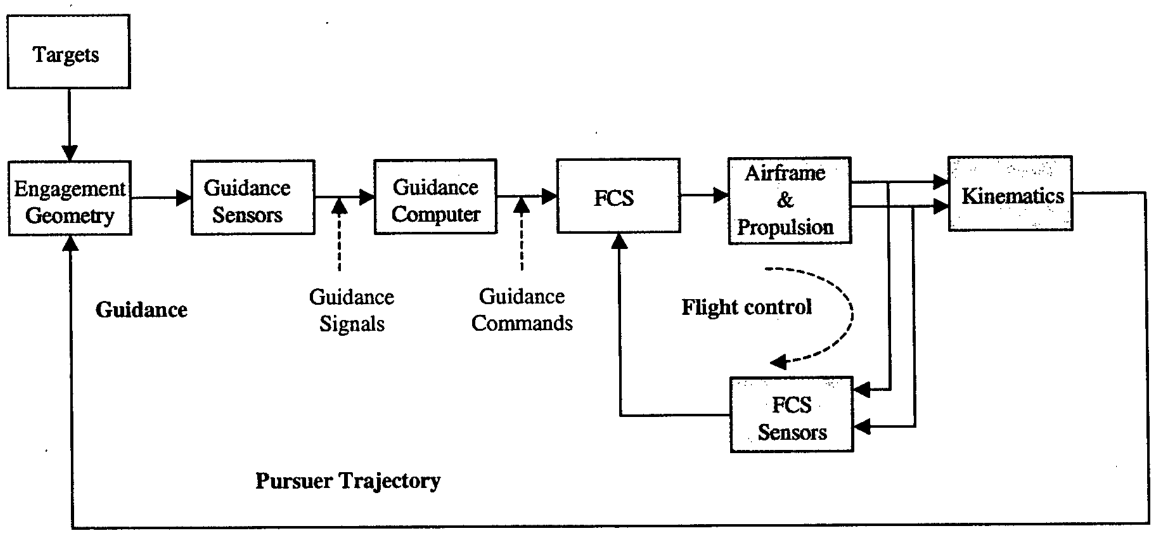

Figure 1 – Missile guidance and control

The dynamics of dual range missiles, heavily depending on flight conditions, may experience significant variations during operation, leading to nonlinearities and uncertainties to the system model. This makes the autopilot design issue much more involved. To get an insight into the complexity of the problem under investigation, the EMRAAT (Extended Medium Range Air-to-Air Technology) missile is considered.

The characteristics of EMRAAT missile were defined in the late 1980's by the U.S. Air Force Armament Laboratory, Eglin Air Force Base, Florida. The 2-D airframe of the EMRAAT missile is illustrated in Figure 2.

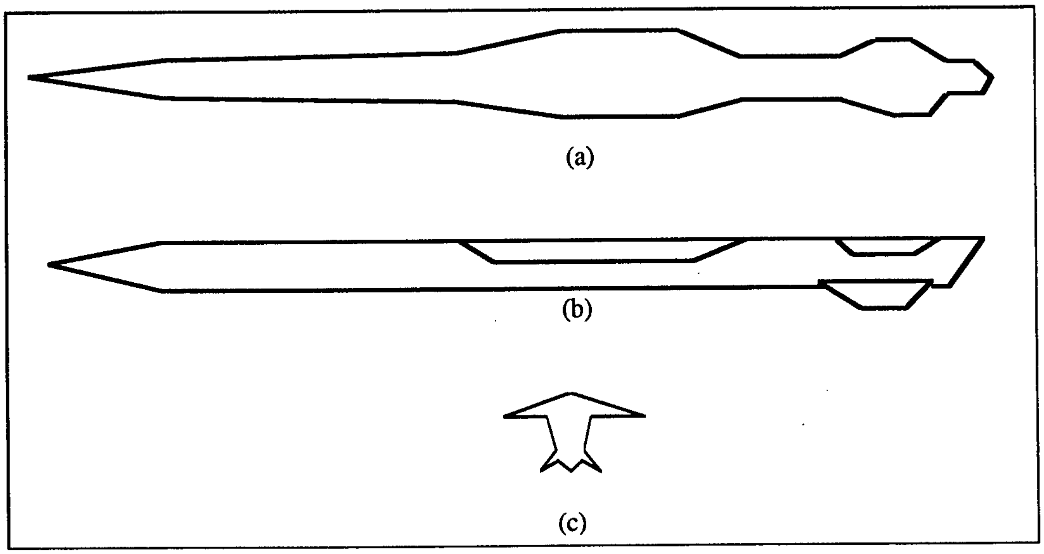

Figure 2 - EMRAAT Airframe. (a) top view, (b) side view and (c) front view

Note that the missile of this type is a longitudinally nonaxi-symmetrical airframe with four tail fins for control which are placed in a non-cruciform. Because of the asymmetric configuration, the missile exhibits many attractive features, such as high-lift, low-drag, air intake, internal carriage, and low-observability. While the asymmetric high-lift configuration gives the missile large maneuverability in its pitch plane, the available load factor in the yaw plane is limited. Therefore, to intercept an incoming target, a bank-to-turn (BTT) steering strategy must be employed. The BTT maneuver is fulfilled through the control of roll and pitch motions. That is, to achieve the desired orientation, a BBT missile rolls the normal plane to the desired direction. The magnitude of the maneuver is controlled by pitch control devices. In contrast to a skip-to-turn autopilot which can be designed without worrying about the coupling among pitch, yaw and roll motions, a BBT autopilot must take into account the Coriolis and gyroscopic couplings due to roll motion. It is also imperative to accommodate the coupling between sideslip, angle of attack and roll rates to achieve rapid and precise response. Furthermore, it should be realized that most of all the aerodynamic coefficients are not constant, but time varying, depending on the flight conditions such as altitude, Mach number and angle of attack, etc, as shown in Figures 3-5. Some of the parameters may change by up to 50% of their nominal values. This is particularly true for dual range missiles. Such a wide range of change demands a highly robust and adaptive control scheme. In this project, the following facts are taken into account in the design of missile autopilot:

- Deflections cannot be adjusted instantaneously due to time delay in actuator dynamics
- Off diagonal element in inertia matrix cannot be assumed to be zero due to asymmetric missile structure
- Aerodynamic parameters can not be treated as slowly time-varying due to varying flight conditions (such as altitude, Mach number, etc.)

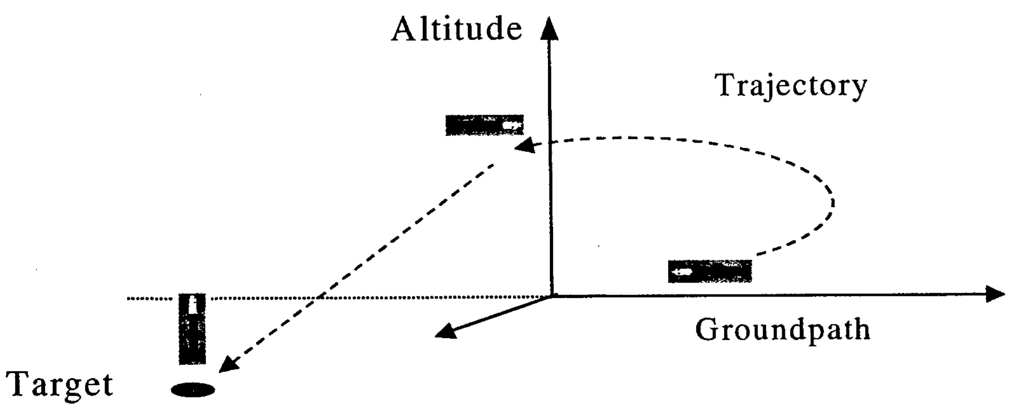

Figure 3 – Dual range missile flying trajectory

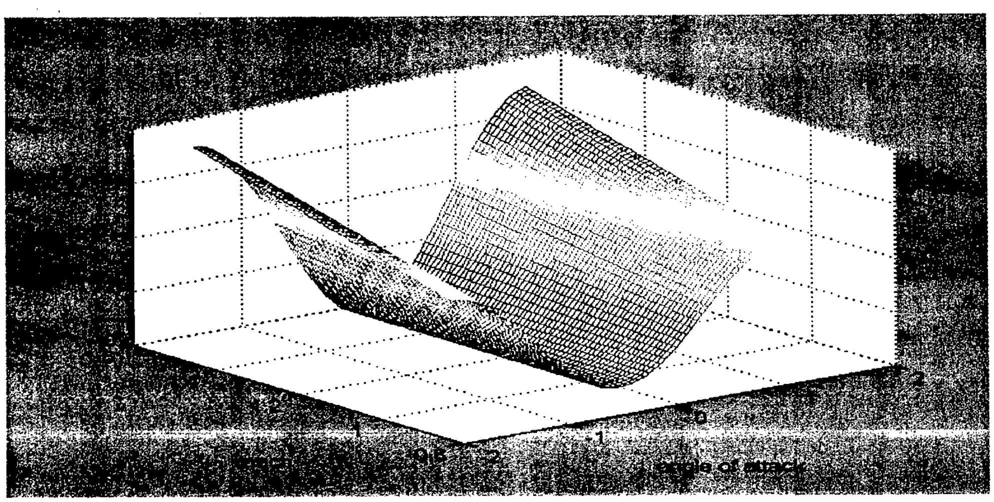

Figure 4- Aerodynamic force coefficient C_N vs. angle of attack and Mach number

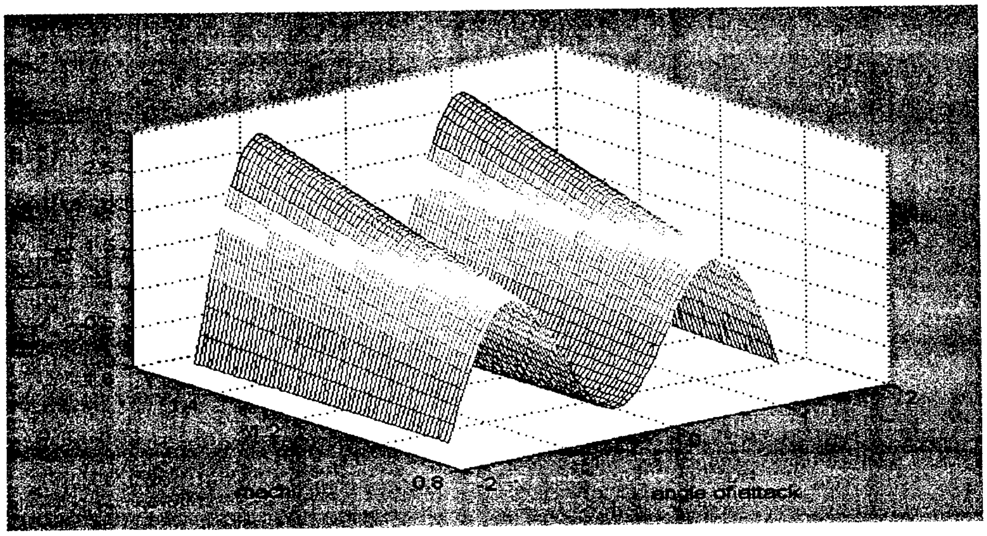

Figure 5 - Aerodynamic force coefficient C_L vs. angle of attack and Mach number

2. Memory-based Control Methodology

In this project, a methodology referred to as the *memory-based* approach for missile flight control was investigated. The fundamental idea behind this approach is somewhat different from traditional methods. More specifically, our control method is not to assume the system is described by a linear model plus perturbations; not to linearize the system; not to estimate certain parameters based on the linear parametric assumption; not to determine the bounds on certain nonlinear functions; not to use infinite switch frequencies; not to involve ad hoc membership functions; not to run the system repeatedly for the same task. Instead, the control scheme is solely based upon certain memorized information such as current system response, previous system response and past control experience. Fundamentally, the desired control signal in the scheme is "learned" and generated from observing and processing the most recent experience stored in a memory. System performance can be continuously improved during system operation. There is no need to repeatedly run the system (a process that is not allowed in missile systems). Another advantage of this approach is that the overall required memory space does not grow with time and is much smaller than most existing methods (Atkeson and Reinkensmeyer 1992, Schaal and Atkeson 1994).

2.1 Information Sets

While there exist many approaches to utilizing memory-based concepts for control systems, our special interest in this project lies in introducing a simple method to build a controller using certain memorized information. To begin, we introduce the following *information sets*

$S_u = \{u_{k-1}, u_{k-2}, \ldots, u_1, u_0\}$ - *control history*

$S_x = \{x_k, x_{k-1}, \ldots, x_1, x_0\}$ - *current and past system responses*

$S_{x^*} = \{x_k^*, x_{k-1}^*, \ldots, x_1^*, x_0^*\}$ - *current and past desired system responses*

Here (and hereafter), u_k, x_k and x^*_k stand for $u(kT)$ -- control signal, $x(kT)$ -- state vector of the system, and $x^*(kT)$ -- the desired trajectory vector, respectively, and T is the sampling period.

Simply speaking, the development of memory-based control algorithms involves two issues: *selecting appropriate memorized information* and *processing the selected information*. There could be many different approaches to addressing these issues, leading to different types of memory-based control algorithms. A typical approach is to make use of all the memorized information and process such information by the "weighted average" method, i.e.

$$u_k = \frac{1}{h}\sum_{i=1}^{h} z_i w_i \tag{1}$$

where

$$z \in R^h \subset S_u \cup S_x \cup S_{x^*}$$

is a vector associated with the stored system information and w_i's represent memory coefficients (*MC*s).

While the memory-based controller as shown in (1) is simple in structure, it suffers from the following drawbacks: 1) the required memory size may become extremely large as time goes by because it makes use of all the past information of the system (h is proportional to k); 2) due to large amounts of information to be processed in (1), extensive computation and lengthy memory search are involved; and 3) there is no guarantee for system stability because the MCs are not derived from system stability consideration.

2.2 Proposed Structure

We now develop a memory-based control scheme in which the above limitations are removed. As a first step, we define the following subsets

$S_u^r \subset S_u$, $S_x^r \subset S_x$ and $S_{x^*}^r \subset S_{x^*}$ such that

$S_u^r = \{u_{k-1}, u_{k-2}, \ldots, u_{k-r}\}$ - *control experience beyond r-step*

$S_x^r = \{x_k, x_{k-1}, \ldots, x_{k-r}\}$ - *current and r-step back system responses*

$S_{x^*}^r = \{x_k^*, x_{k-1}^*, \ldots, x_{k-r}^*\}$ - *current and r-step back desired system responses*

Here $1 \leq r << k$ is an integer. These sets form the database of experience which are retrieved and processed via

$$u_k = \phi(z_r; w_r) \tag{2}$$

where

$$z_r \in R^p \subset S_u^r \oplus S_x^r \oplus S_{x^*}^r$$

and $\phi(.)$ is a mapping function that converts the selected information z_r into u_k (through certain MCs). The difference between (1) and (2) is that only the most recent experiences are honored in (2) while the past information beyond *r*-step is forgotten. This is motivated by the fact that for a practical system the latest system statues have more influence on its future behavior. Moreover, the selected information is processed through the mapping function $\phi(.)$. Figure 6 illustrates the construction of the first order memory-based controller and Figure 7 is a schematic diagram for memory-based control with memorized information up to step *r*.

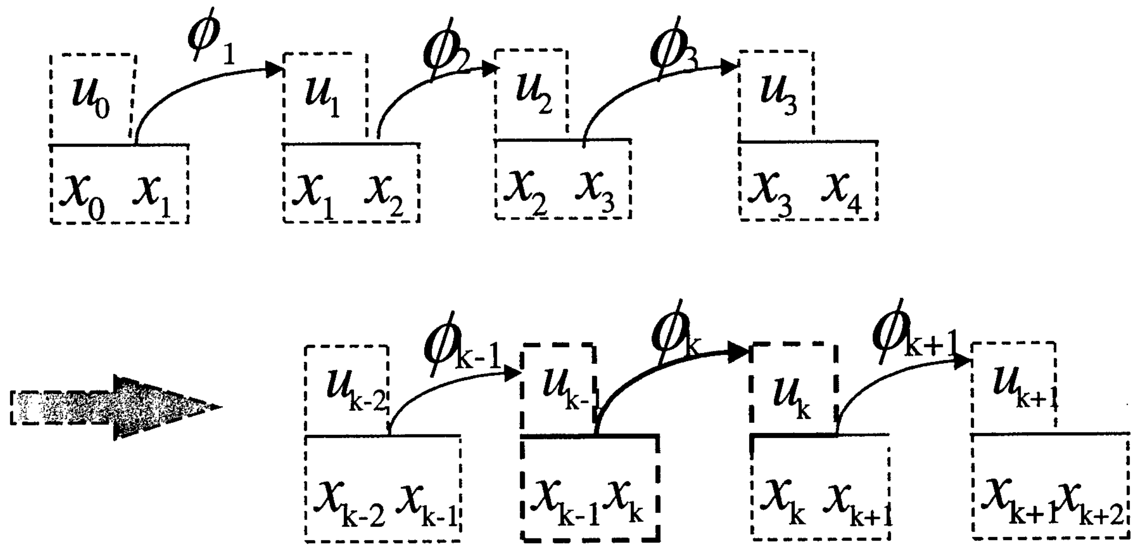

Figure 6 - Construction of 1st order memory-based control

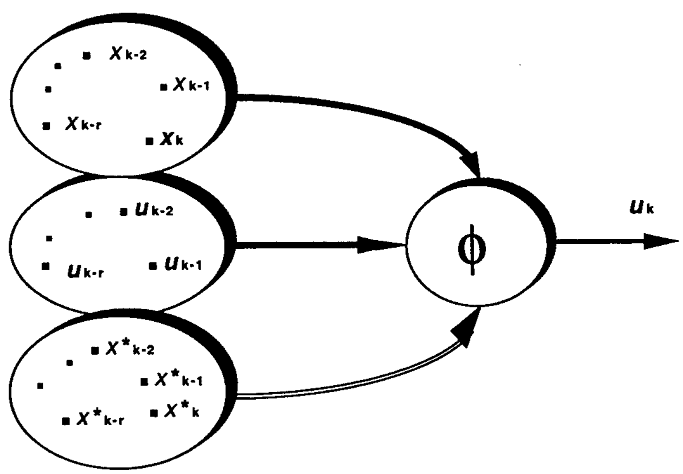

Figure 7 - The r^{th}-order memory-based control scheme

One of the favorable advantages of the proposed method is that the required memory size does not grow with time and is much smaller as compared with other methods (e.g., CMAC (Albus, 1975) and ACAM (Atkeson et al 1992)). To make this point clearer, let us consider building a first order memory-based controller. Essentially in this case we only need the following information:

$$S_u^1 = \{u_{k-1}\}$$

$$S_x^1 = \{x_{k-1}, x_k\}$$

and

$$S_{x^*}^1 = \{x_{k-1}^*, x_k^*\}$$

which is processed/generalized as in Figure 8. Similarly, to construct a second order memory-based controller we only need the stored information (Figure 9)

$$S_u^2 = \{u_{k-1}, u_{k-2}\}$$

$$S_x^2 = \{x_{k-2}, x_{k-1}, x_k\}$$

and

$$S_{x^*}^2 = \{x_{k-2}^*, x_{k-1}^*, x_k^*\}$$

As can be seen, memory space is not an issue here because the required memory does not grow with time.

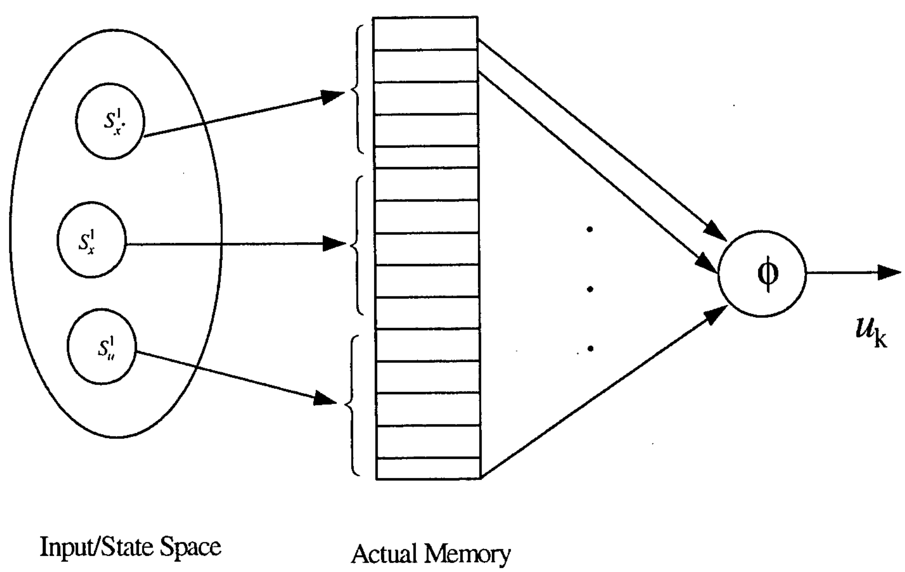

Figure 8 - A first-order memory-based control

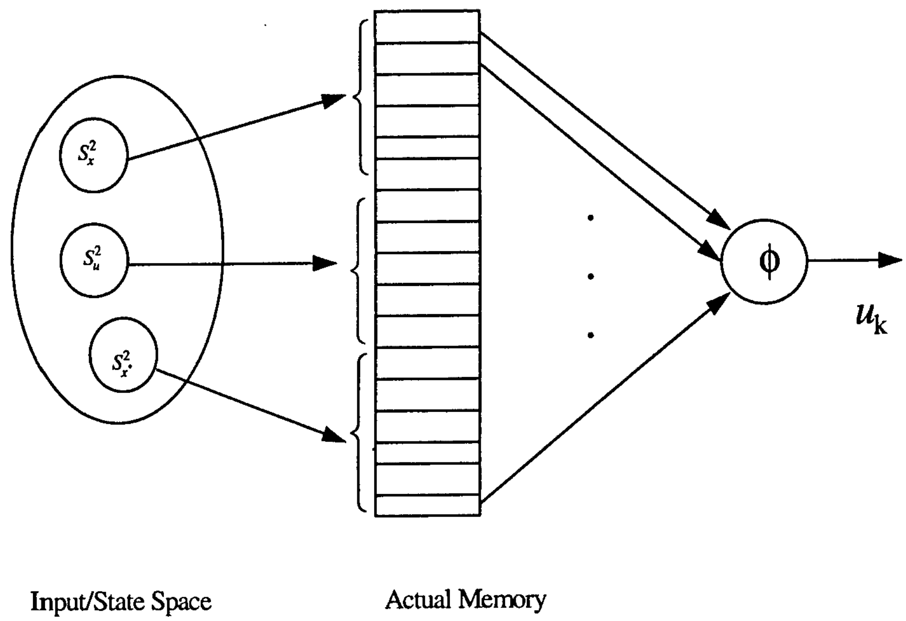

Figure 9 - A second-order memory-based controller

Regarding the memory-based controller (2), if we view the memory-based controller as a *mechanism* (*network*) that *on-line* processes the memorized information and produces a control action at time instant t = kT, then the design of a memory-based controller boils down to specify such a mechanism, i.e., specifying the mapping function $\phi(.)$ and the memory coefficients (MCs) w_r. In this project, we use the following mapping function, among others,

$$\phi(\sigma, y) = \frac{2}{\sigma}\left(\frac{1-e^{-\sigma y}}{1+e^{-\sigma y}}\right), \quad \sigma > 0$$

It is interesting to note that such a mapping function exhibits two salient properties

i) $\quad |\phi(\sigma, y)| \leq \dfrac{2}{\sigma}$

ii) $\quad \lim_{\sigma \to 0} \phi(y, \sigma) \to y$

The first property allows us to adjust the magnitude of the control signal by choosing a proper value of σ and the second property implies that $\phi(.)$ behaves like a linear function as σ tends to a small number. This property is useful in our stability analysis.

3. Dynamic Modeling and Autopilot Design Problem Statement

Using the standard body-fixed coordinate system [1], [7] (i.e., the origin is at the center of mass of the missile, the positive x-axis extends forward through the nose of the missile, the positive z-axis extends downward, and the y-axis completes the right-handed triad), the following motion equations can be established

$$J\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix}+\begin{bmatrix}p\\ q\\ r\end{bmatrix}\times J\begin{bmatrix}p\\ q\\ r\end{bmatrix}=QSd\left\{\begin{bmatrix}n_1(\beta,p,r;c)\\ n_2(\alpha,\dot{\alpha},q;c)\\ n_3(\beta,p,r;c)\end{bmatrix}+C(c)\begin{bmatrix}\delta_p\\ \delta_q\\ \delta_r\end{bmatrix}\right\} \tag{3}$$

with

$$J=\begin{bmatrix}J_{xx} & -J_{xy} & -J_{xz}\\ -J_{xy} & J_{yy} & -J_{yz}\\ -J_{xz} & -J_{yz} & J_{zz}\end{bmatrix},\quad C(c)=\begin{bmatrix}C_{l_{\delta_p}} & 0 & C_{l_{\delta_r}}\\ 0 & C_{m_{\delta_q}} & 0\\ C_{n_{\delta_p}} & 0 & C_{n_{\delta_r}}\end{bmatrix}$$

and

$$\begin{bmatrix}n_1(\beta,p,r;c)\\ n_2(\alpha,\dot{\alpha},q;c)\\ n_3(\beta,p,r;c)\end{bmatrix}=\begin{bmatrix}C_{l_\beta}\beta+C_{l_p}p+C_{l_r}r\\ C_{m_\alpha}\alpha+C_{m_{\dot{\alpha}}}\dot{\alpha}+C_{m_q}q\\ C_{n_\beta}\beta+C_{n_p}p+C_{n_r}r\end{bmatrix}$$

The definition of each variable can be found in [1]. As explicitly reflected in the above equations, the yaw-pitch-roll motion depends on aerodynamic coefficients/parameters (denoted by c here). The dynamic motions of the missile also depend on angle of sideslip (β) and angle of attack (α) as defined by

$$\dot{\alpha} = q - \tan(\beta)[p\cos(\alpha) - r\sin(\alpha)] + \frac{g}{V\cos(\beta)}(\cos(\alpha)\cos(\phi) + \sin(\alpha)\sin(\theta))$$

$$+ \frac{gQS}{WV\cos(\beta)}(C_{N_\alpha}\alpha + C_{N_q}q + C_{N_{\delta_q}}\delta_q)\cos(\alpha)$$

$$\dot{\beta} = p\sin(\alpha) - r\cos(\alpha) + \frac{gQS}{WV}(C_{Y_\beta}\beta + C_{Y_p}p + C_{Y_r}r + C_{Y_{\delta_p}}\delta_p + C_{Y_{\delta_r}}\delta_r)\cos(\beta)$$

$$+ \frac{g}{V}\cos(\theta)\sin(\phi)\cos(\beta)$$

Also note that the diagonal elements of the inertia matrix of the EMRAAT missile under investigation are $J_{xx} = 1.08\ slug * ft^2$, $J_{yy} = 70.13\ slug * ft^2$ and $J_{zz} = 70.66\ slug * ft^2$. It is seen that the first diagonal element, J_{xx}, of the inertia matrix is not "dominant" as compared with its off-diagonal elements ($J_{xy} = J_{yx} = 0.274 slug * ft^2$). Physically, this implies that the motions in yaw, pitch and roll are not decoupled. Consequently, the diagonal assumption of the inertia matrix commonly used does not hold for the EMRAAT missiles. Furthermore, the surface deflections δ_p, δ_q and δ_r cannot be adjusted directly and instantaneously due to time delay in surface dynamics. Therefore, the actuator dynamics must be included in the model.

In this work we do not assume that the inertia matrix is diagonal. Moreover, we consider the following model to reflect a delay and nonlinear relationship between the control input command and its realization:

$$\dot{\delta}_p = \varphi_p(\delta_p) + b_p v_p \tag{4a}$$

$$\dot{\delta}_q = \varphi_q(\delta_q) + b_q v_q \tag{4b}$$

$$\dot{\delta}_r = \varphi_r(\delta_r) + b_r v_r \tag{4c}$$

where v_p, v_q and v_r are the control inputs for yaw, pitch and roll motions, respectively, b_p, b_q and b_r are constant control gains, and φ_p, φ_q and φ_r are nonlinear functions. The missile autopilot design problem to be addressed is as follows:

Given a set of commands in terms of yaw, pitch and roll angle $(\phi^*, \theta^*, \psi^*)$ *and its velocity* $(\dot{\phi}^*, \dot{\theta}^*, \dot{\psi}^*)$, *design the control input signals* v_p, v_q and v_r *to automatically adjust the surface deflections such that the actual yaw, pitch and roll motions of the missile track the desired motions closely.*

In practice, the desired orientation commands $(\phi^*, \theta^*, \psi^*)$ are provided by the guidance unit (Figure 1). The function of the autopilot unit is to steer the missile to track the given path. In this work, we assume that the guidance commands $(\phi^*, \theta^*, \psi^*)$ are available for control design.

4. Autopilot for Orientation Tracking

For later development, the following variables are defined:

$$\Theta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad n(.,c) = \begin{bmatrix} n_1(.,c) \\ n_2(.,c) \\ n_3(.,c) \end{bmatrix}, \quad u = \begin{bmatrix} \delta_p \\ \delta_q \\ \delta_r \end{bmatrix}$$

Note that Θ and ω share the relation [7]

$$\omega = A(\theta,\phi)\dot{\Theta} \quad \text{and} \quad \dot{\omega} = \dot{A}\dot{\Theta} + A\ddot{\Theta} \tag{5}$$

where

$$A(\theta,\phi) = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix}$$

which is non-singular as long as $\theta \neq \pm\pi/2$. The case that A becomes singular is known as *gimbal lock*. In our analysis, we assume such unusual flight/launch condition does not occur. If the missile under consideration may experience the gimbal lock, a different rotation order may be employed. With the above notations, the aerodynamics and the actuator dynamics can be expressed as

$$\ddot{\Theta} = f(.,c) + g(.,c)u \tag{6}$$

$$\dot{u} = \varphi(u) + bV \tag{7}$$

where

$$f(.,c) = (JA)^{-1}\{QSdn(.,c) - J\dot{A}\dot{\Theta} - (A\dot{\Theta}) \times J(A\dot{\Theta})\} \tag{7a}$$

$$g(.,c) = (JA)^{-1} QSdC(c) \tag{7b}$$

$$V = [v_p \; v_q \; v_r]^T, \; \varphi = [\varphi_p \; \varphi_q \; \varphi_r]^T, \;\; b = diag(b_p, b_q, b_r) \tag{7c}$$

To describe the control strategy, let us define the orientation tracking error

$$e = \Theta - \Theta^* \tag{8}$$

and filtered tracking error

$$s = \dot{e} + k_0 e \tag{9}$$

where $k_0 > 0$ is a constant chosen by the designer.

4.1 Inverse Dynamics Based Autopilot

Theorem 1

Considering the missile modeled by (6)-(7). If the control input V is generated by

$$V = B_I^{-1}(-k_z z - A_I) \tag{10}$$

where

$$z = f(\dot{\Theta}, \Theta) + g(\Theta)u - \ddot{\Theta}^* + k_s s + k_0 \dot{e} \tag{11}$$

$$\begin{aligned} A_I(.) = {} & \frac{\partial f}{\partial \Theta}\dot{\Theta} + \left(\frac{\partial g}{\partial \Theta}\dot{\Theta}\right)u + \left(\frac{\partial f}{\partial \dot{\Theta}} + (k_0 + k_s)I\right)(f + gu) \\ & + g\varphi - \dddot{\Theta}^* - (k_s + k_0)\ddot{\Theta}^* + k_0 k_s \dot{e} \end{aligned} \tag{12a}$$

$$B_I(.) = gb \tag{12b}$$

and $k_s > 0$ and $k_z > 0$ are design constants, then asymptotic orientation tracking is ensured, i.e., $\Theta \to \Theta^*$ and $\dot{\Theta} \to \dot{\Theta}^*$ as $t \to \infty$.

Proof:

With s and z as defined in (9) and (11), it can be shown that

$$\dot{s} = -k_s s + z \tag{13}$$

where $k_s > 0$ is a design constant. Taking derivative of z with respect to time and using (6)-(7) yields

$$\dot{z} = A_I(.) + B_I(.)V \tag{14}$$

where A_I and B_I are defined as in (12). Under the control of V as given in (10), it is readily verified that

$$\dot{z} = -k_z z \tag{15}$$

Therefore the closed-loop dynamics of the system are governed by (9), (13) and (15), which can be expressed compactly as

$$\frac{d}{dt}\begin{pmatrix} e \\ s \\ z \end{pmatrix} = \begin{bmatrix} -k_0 I & I & 0 \\ 0 & -k_s I & I \\ 0 & 0 & -k_z I \end{bmatrix}\begin{bmatrix} e \\ s \\ z \end{bmatrix} \tag{16}$$

where $I \in R^{3\times 3}$ is a unit matrix. It is seen that the augmented system has the following eigenvalues: $\lambda_{ei} = -k_0$, $\lambda_{si} = -k_s$ and $\lambda_{zi} = -k_z$ $(i = 1, 2, 3)$. Therefore it can be concluded that e, s (hence $\dot{e}$) and z are asymptotically stable, i.e., $\|z\| \to 0$, $\|e\| \to 0$ and $\|\dot{e}\| \to 0$ as $t \to \infty$. The result as stated in the theorem is then established.

4.2 Autopilot with Memory-based Compensation

It should be noted that for dual range missiles system parameters may undergo significant variation due to the wide change of flight condition [8]-[12]. An example is shown in Figure 10, where C_D and C_L are missile aerodynamic force coefficients [13]. They are computed as a function of Reynolds number, angle of attack, and Mach number. It can be seen from Figure 10 that as the angle of attack changes, C_D and C_L vary significantly, which leads to unknown and varying system parameters. To reflect this fact, we consider the general system parameter c in the form

$$c = c_0 + \delta c(t) \tag{17}$$

and express the dynamic motions of equation as

$$\ddot{\Theta} = f_0(.,c_0) + g_0(.,c_0)u + \Delta_1(\Theta, \dot{\Theta}, u, \delta c) \tag{18a}$$

$$\dot{u} = \varphi_0(u) + \Delta_\varphi(.) + bV \tag{18b}$$

where f_0 and g_0 are the nominal functions associated with the available system modeling information whereas Δ_1 represents the unknown portion of the system due to unknown flight conditions. As a result the previously developed control algorithms are not applicable.

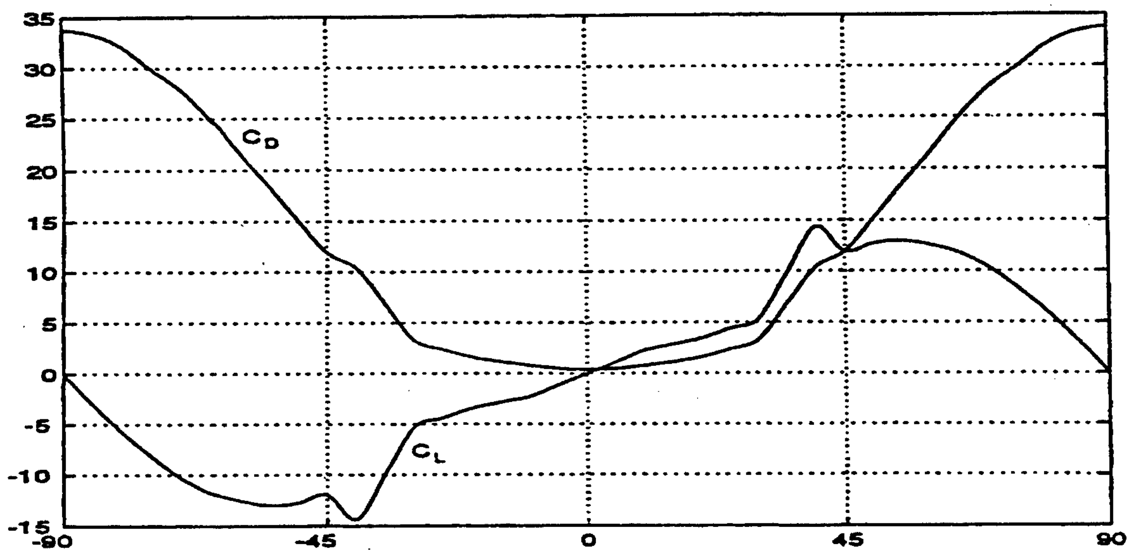

Figure 10 - Aerodynamic force Coefficients vs. angle of attack

It is therefore of practical interest to develop the corresponding control scheme that achieves acceptable performance under varying flight conditions. In this work, it is proposed to combine the memory-based compensation with partial inverse dynamics control to achieve this objective. As a first step, we rewrite (18a) as

$$\dot{s} = -k_s s + z_0 + \Delta_1 \tag{19}$$

where

$$z_0 = f_0(\dot{\Theta},\Theta) + g_0(\Theta)u - \ddot{\Theta}^* + k_0\dot{e} + k_s s \tag{20}$$

and e and s are defined as before. Taking derivative of z_0 with respect to time and using (18a)-(18b) gives

$$\dot{z}_0 = A_{I0} + B_{I0}V + \Delta_2 \tag{21}$$

where

$$A_{I0} = \frac{\partial f_0}{\partial \Theta}\dot{\Theta} + \left(\frac{\partial g_0}{\partial \Theta}\dot{\Theta}\right)u + \left(\frac{\partial f_0}{\partial \dot{\Theta}} + (k_0 + k_s)I\right)(f_0 + g_0 u) + g_0\varphi - \dddot{\Theta}^* - (k_0 + k_s)\ddot{\Theta}^* + k_s k_0 \dot{e} \tag{22a}$$

$$B_{I0} = g_0 b \tag{22b}$$

$$\Delta_2 = \left(\frac{\partial f_0}{\partial \dot{\Theta}} + (k_0 + k_s)I\right)\Delta_1 + g_0\Delta_\varphi \tag{23}$$

Note that the control scheme previously developed is fairly complicated in that it involves extensive analytical calculation. Especially, because of the complexity of f_0, it demands significant manipulation to analytically find A_{I0} (this would be painful when flight conditions vary). Furthermore, the control scheme needs $\dddot{\Theta}^*$ to be available, which may require a complicated guidance unit in the system. In order to simplify on-line computations in guidance and control, A_{I0} (and therefore $\dddot{\Theta}^*$) is not used in our control scheme. Instead, we treat A_0 as part of the lumped system uncertainty as expressed below

$$\tilde{\Delta} = A_{I0} + \Delta_2 \tag{24a}$$

For such a lumped uncertainty, we consider two cases:

- *Case I - $\tilde{\Delta}$ is unknown and slowly time varying, i.e.,*

$$\sup_{t\geq 0}\left\|\frac{d\tilde{\Delta}}{dt}\right\| = \nu_1 \approx 0 \tag{24b}$$

- *Case 2 - $\tilde{\Delta}$ is unknown and fast time-varying in that*

$$\sup_{t\geq 0}\left\|\frac{d\tilde{\Delta}}{dt}\right\| = \nu_1 < \infty \tag{24c}$$

In addition, we assume that there exists a constant ε_1 such that

$$\|\Delta_1\| \leq \varepsilon_1 < \infty \tag{24d}$$

Remark

For a missile undergoing no abrupt change of flight condition, it seems reasonable to make the above assumption (24b) or (24c) on $\tilde{\Delta}$ because its variation cannot be infinitely fast. The condition of (24d) is also necessary for the system to admit a feasible control strategy.

The proposed control scheme is given as follows

$$V = B_{I0}^{-1}(-k_z z_0 - V_m) \tag{25}$$

where V_m denotes the memory-based compensation. For simplicity, we consider the following first order memory-based control,

$$V_m(k) = \frac{2}{\sigma}\left(\frac{1-e^{\sigma y}}{1+e^{-\sigma y}}\right) \tag{26a}$$

with

$$y = w_0 V_m(k-1) + w_1 z_0(k) + w_2 z_0(k-1) \tag{26b}$$

where w_0, w_1, and w_2 are the memory coefficients to be determined. It is seen that only one step back control history and system response are needed in the scheme.

Theorem 2

Consider the missile model with lumped uncertainty due to varying flight conditions as described in by (18) and (24). Let the control input be generated by (25) with the memory-based compensation (26a). If the memory coefficients are given by

$$w_0 = 1, \quad w_1 = (2 - Tk_z)/T, \quad w_2 = (Tk_z - 1)/T \tag{26c}$$

where T is the sampling period, then the missile orientation tracks the desired orientation with bounded error.

Proof:

We first prove that $\|z_0\|$ is bounded. With the control of (25), it follows from (21) that

$$\dot{z}_0 = -k_z z_0 + \tilde{\Delta} - V_m \tag{27}$$

For a small sampling period T, using Euler formula (27) can be expressed as

$$z_0(k+1) = (1 - k_z T) z_0(k) + T(\tilde{\Delta}(k) - V_m(k)) \tag{28}$$

With one step back time shift, we get

$$z_0(k) = (1 - k_z T) z_0(k-1) + T(\tilde{\Delta}(k-1) - V_m(k-1)) \tag{29}$$

Subtracting (29) from (28) gives

$$\begin{aligned} z_0(k+1) &= (2 - k_z T) z_0(k) + (k_z T - 1) z_0(k-1) \\ &\quad + T(\tilde{\Delta}(k) - \tilde{\Delta}(k-1)) - T(V_m(k) - V_m(k-1)) \end{aligned} \tag{30}$$

Upon using (26a)-(26c) and using the property of $\phi(.)$ with σ small enough, it can be shown that

$$z_0(k+1) = T(\tilde{\Delta}(k) - \tilde{\Delta}(k-1)) \tag{31}$$

Note that for a sufficiently small sampling period T,

$$\left\|\tilde{\Delta}(k) - \tilde{\Delta}(k-1)\right\| \le T \sup_{t \ge 0} \left\| \frac{d\tilde{\Delta}}{dt} \right\| = T\nu_1$$

Therefore, it is seen from (31) that under the conditions (24b)

$$\left\| z_0(k+1) \right\| = 0 \quad \forall k \ge 1$$

and under the condition (24c)

$$\left\| z_0(k+1) \right\| \le T^2 \nu_1 < \infty \quad \forall k \ge 1$$

Thus for both cases it can be concluded that $\|z_0\|$ is bounded. In view of (19) and the assumption (24d), it is readily established that $\|s\|$, and therefore, $\|e\|$ and $\|\dot{e}\|$ are bounded.

Remarks

1) It is interesting to note that the proposed memory-based control turns out to be fairly simple as compared to the one developed previously. One only needs to select the control parameters k_0, k_s and k_z.

2) Also note that the proposed scheme is essentially a partial inverse dynamics control plus a memory-based control. There is no need to carry out the rather complicated manipulations/procedures to analytically find A_{I0}, only B_{I0} is used, which makes the design and real-time implementation more feasible.

3) Also note that since the control algorithm does not involve ν_1 or ε_1, no estimation of such parameters is needed.

5. Autopilot for Position and Orientation Tracking

The previous control algorithms are designed for pure missile orientation control. In this section the algorithms are extended to a more general case where a rigid body missile undergoes both position and orientation maneuvers simultaneously. In such a case, the dynamic behavior of the missile can be characterized by [7],

$$\ddot{P} = \frac{1}{m} R f^b \tag{32a}$$

$$\ddot{\Theta} = (JA)^{-1}\left(\tau^b - A\dot{\Theta} \times JA\dot{\Theta} - J\dot{A}\dot{\Theta}\right) \tag{32b}$$

where $P = [p_x \; p_y \; p_z]^T$ is the position vector of the center of mass; $\Theta = [\phi \; \theta \; \psi]^T$ is the vector of yaw, pitch, and roll Euler angles; $R(\phi, \theta, \psi)$ is the 3×3 rotation matrix; $A(\theta, \phi)$ is a 3×3 matrix; J is the constant moment of inertia matrix; $f^b = [f_x^b \; f_y^b \; f_z^b]^T$ is the 3×1 force vector in the body frame; and $\tau^b = [\tau_x^b \; \tau_y^b \; \tau_z^b]^T$ is the 3×1 torque vector in the body frame. Note that f^b and τ^b are to be controlled by the fin kinematics, such as flapping frequency n, the maximum stroke angle, the angle of attack, and fin rotation time, etc. To reflect this fact, we denote f^b and τ^b as

$$\begin{bmatrix} f^b \\ \tau^b \end{bmatrix} = N(u)$$

where u is a virtual control variable. Since u cannot be adjusted directly, the following actuator dynamics

$$\dot{u} = \varphi(u) + bV \tag{33}$$

is included as part of the system model, where b is the control influence gain matrix and V is the control input. The control problem under investigation is stated as follows: Design V such that the missile flying position and orientation (P, Θ) closely track the guidance commands (P^*, Θ^*). Mathematically, the task is to derive algorithm for V so that

$$\left\| \begin{matrix} P - P^* \\ \Theta - \Theta^* \end{matrix} \right\| \to 0 \text{ as } t \to \infty \quad \text{or} \quad \left\| \begin{matrix} P - P^* \\ \Theta - \Theta^* \end{matrix} \right\| \le \varepsilon \text{ as } t \to \infty$$

5.1 Inverse Dynamics Based Autopilot for Position and Orientation Tracking

Considering both position and orientation, (32) can be expressed as

$$\ddot{Y} = h(\dot{Y}, Y) + M(Y, u) \tag{34}$$

where

$$Y = [P \; \Theta]^T \tag{35}$$

$$h(\dot{Y}, Y) = \begin{bmatrix} 0 \\ (JA)^{-1}\left(-A\dot{\Theta} \times JA\dot{\Theta} - J\dot{A}\dot{\Theta}\right) \end{bmatrix} \tag{36}$$

and

$$M(Y, u) = \begin{bmatrix} \dfrac{R}{m} & 0 \\ 0 & (JA)^{-1} \end{bmatrix} N(u) \tag{37}$$

Define the tracking error (e) and the filtered tracking error (s) by

$$e = Y - Y^* \tag{38}$$

and

$$s = \dot{e} + k_0 e \tag{39}$$

respectively, where $k_0 > 0$ is a design constant. Taking derivative of s with respect to time yields

$$\dot{s} = h(\dot{Y}, Y) + M(Y, u) - \ddot{Y}^* + k_0 \dot{e} \tag{40}$$

where the equation (34) has been used. Virtually, we need to design u to achieve the tracking objective. However, since u enters into the system in a nonlinear way, u cannot be designed directly. In fact, u is governed by the actuator dynamics (33), which can be influenced by adjusting the control variable V. To this end, we rewrite (40) as

$$\dot{s} = -k_s s + z \tag{41a}$$

where

$$z = h(\dot{Y}, Y) + M(Y, u) - \ddot{Y}^* + k_s s + k_0 \dot{e} \tag{41b}$$

and $k_s > 0$ is a design constant. It is seen from (41a) that if z tends to zero as $t \to \infty$, then $s \to 0$. Consequently it follows from (39) that $\dot{e} \to 0$ and $e \to 0$ as $t \to \infty$. Therefore, we focus on stabilizing z in what follows. Taking derivative of z with respect to time, gives

$$\dot{z} = \frac{\partial h}{\partial Y}\dot{Y} + \frac{\partial h}{\partial \dot{Y}}\ddot{Y} + \frac{\partial M}{\partial Y}\dot{Y} + \frac{\partial M}{\partial u}\dot{u} - \dddot{Y}^* + k_s \dot{s} + k_0 \ddot{e} \tag{42}$$

Using (33), (34) and (40), we can further express (42) as

$$\dot{z} = A_{II}(.) + B_{II}(.)V \tag{43a}$$

where

$$\begin{aligned} A_{II}(.) = {} & \left(\frac{\partial h}{\partial Y} + \frac{\partial M}{\partial Y}\right)\dot{Y} + \left(\frac{\partial h}{\partial \dot{Y}} + (k_s + k_0)I\right)(h + M) \\ & + \frac{\partial M}{\partial u}\varphi(u) - \dddot{Y}^* - (k_s + k_0)\ddot{Y}^* + k_s k_0 \dot{e} \end{aligned} \tag{43b}$$

$$B_{II}(.) = \frac{\partial M}{\partial u} b \tag{43c}$$

Now the control input V is designed such that

$$V = B_{II}^{-1}\left(-k_z z - A_{II}\right) \tag{44}$$

where $k_z > 0$ is a design constant. It can be verified that such a controller leads to

$$\dot{z} = -k_z z \tag{45}$$

i.e., $z \to 0$ as $t \to \infty$. To summarize, we have the following theorem.

Theorem 3

Considering the missile dynamics described by (32). If the control input V is given by (44), where z is calculated by (41b), and A_{II} and B_{II} are determined as in (43b) and (43c), then both P and Θ track the guidance commands P^* and Θ^* asymptotically.

Proof:

With the proposed control, it can be shown that the closed-loop system dynamics become

$$\dot{e} = -k_0 e + s$$

$$\dot{s} = -k_s s + z$$

$$\dot{z} = -k_z z$$

which can be expressed compactly as

$$\frac{d}{dt}\begin{pmatrix} e \\ s \\ z \end{pmatrix} = \begin{bmatrix} -k_0 I & I & 0 \\ 0 & -k_s I & I \\ 0 & 0 & -k_z I \end{bmatrix}\begin{bmatrix} e \\ s \\ z \end{bmatrix}$$

where $I \in R^{6\times6}$ is a unit matrix. It is seen that the augmented system has the following eigenvalues: $\lambda_{ei} = -k_0$, $\lambda_{si} = -k_s$, $\lambda_{zi} = -k_z$ $(i = 1,2,\ldots,6)$. Therefore it is concluded that

e, $\dot{e}$ and z are asymptotically stable, i.e., $\|z\| \to 0$, $\|e\| \to 0$ and $\|\dot{e}\| \to 0$ as $t \to \infty$. The result as stated in the theorem is then established.

5.2 Autopilot with Memory-based Compensation

Due to varying flight condition, the aerodynamics and actuator dynamics of dual range missiles become

$$\ddot{Y} = h_0(\dot{Y}, Y; c_0) + M_0(Y, u) + \Delta_{hM}(\delta c, Y, \dot{Y}, u) \tag{46}$$

and

$$\dot{u} = \varphi_0(u) + bV + \Delta_\varphi(u, t) \tag{47}$$

In such a case the control algorithms developed in Section 5.1 are not applicable. It is interesting to note that the memory-based control previously developed for orientation tracking can be extended to cover this situation. To show this, we rewrite (46) as

$$\dot{s} = -k_s s + z_0 + \Delta_{hM} \tag{48}$$

where

$$z_0 = h_0(\dot{Y}, Y) + M_0(Y, u) - \ddot{Y}^* + k_0\dot{e} + k_s s \tag{49}$$

Taking derivative of z_0 with respect to time and using (46)-(47), we have

$$\dot{z}_0 = B_{II0}V + (A_{II0} + \Delta_2) \tag{50}$$

where

$$A_{II0} = \left(\frac{\partial h_0}{\partial Y} + \frac{\partial M_0}{\partial Y}\right)\dot{Y} + \left(\frac{\partial h_0}{\partial \dot{Y}} + (k_0 + k_s)I\right)(h_0 + M_0)$$
$$+ \frac{\partial M_0}{\partial u}\varphi_0 - \dddot{Y}^* - (k_0 + k_s)\ddot{Y}^* + k_s k_0 \dot{e} \tag{51a}$$

$$B_{II0} = \frac{\partial M_0}{\partial u} b \tag{51b}$$

$$\Delta_2 = \left(\frac{\partial h_0}{\partial \dot{\Theta}} + (k_0 + k_s)I\right)\Delta_{hM} + \frac{\partial M_0}{\partial u}\Delta_\varphi \tag{52}$$

The proposed memory-based control is of the form

$$V = B_{II0}^{-1}(-k_z z_0 - V_m) \tag{53}$$

where V_m is generated by a memory-based unit to be specified. Note that to simplify the design procedure and on-line computation, A_{II0} is not used as part of the control input. The effect of A_{II0}, together with that of Δ_2, is to be compensated by V_m. Let $\tilde{\Delta} = A_{II0} + \Delta_2$ be the lumped uncertainty of the system. Since $\tilde{\Delta}$ is unknown and time-varying, the proposed control only uses the following information on $\tilde{\Delta}$:

- *Case I - $\tilde{\Delta}$ is unknown and slowly time-varying such that*

$$\left\| \frac{d\tilde{\Delta}}{dt} \right\| = \nu_2 \approx 0 \tag{54a}$$

- *Case II - $\tilde{\Delta}$ is unknown and time-varying such that*

$$\left\| \frac{d\tilde{\Delta}}{dt} \right\| = \nu_2 < \infty \tag{54b}$$

In addition, we assume that $\|\Delta_{hM}\|$ is bounded by a constant in that

$$\|\Delta_{hM}\| \le \varepsilon_2 < \infty \tag{54c}$$

Now we are ready to state the following result.

Theorem 4

Consider the missile model as described by (46)-(47) with lumped uncertainty satisfying (54). If the control input is generated by (53) with the memory-based compensation

$$V_m(k) = \frac{2}{\sigma}\left(\frac{1-e^{\sigma y}}{1+e^{-\sigma y}}\right) \tag{55a}$$

$$y = w_0 V_m(k-1) + w_1 z_0(k) + w_2 z_0(k-1) \tag{55b}$$

where the memory coefficients w_0, w_1, and w_2 are given by

$$w_0 = 1, \quad w_1 = (2 - Tk_z)/T, \quad w_2 = (Tk_z - 1)/T \tag{55b}$$

then the position and orientation of the missile track the guidance commands with bounded error.

Proof:

The result can be justified following the same procedures as in the proof of Theorem 2.

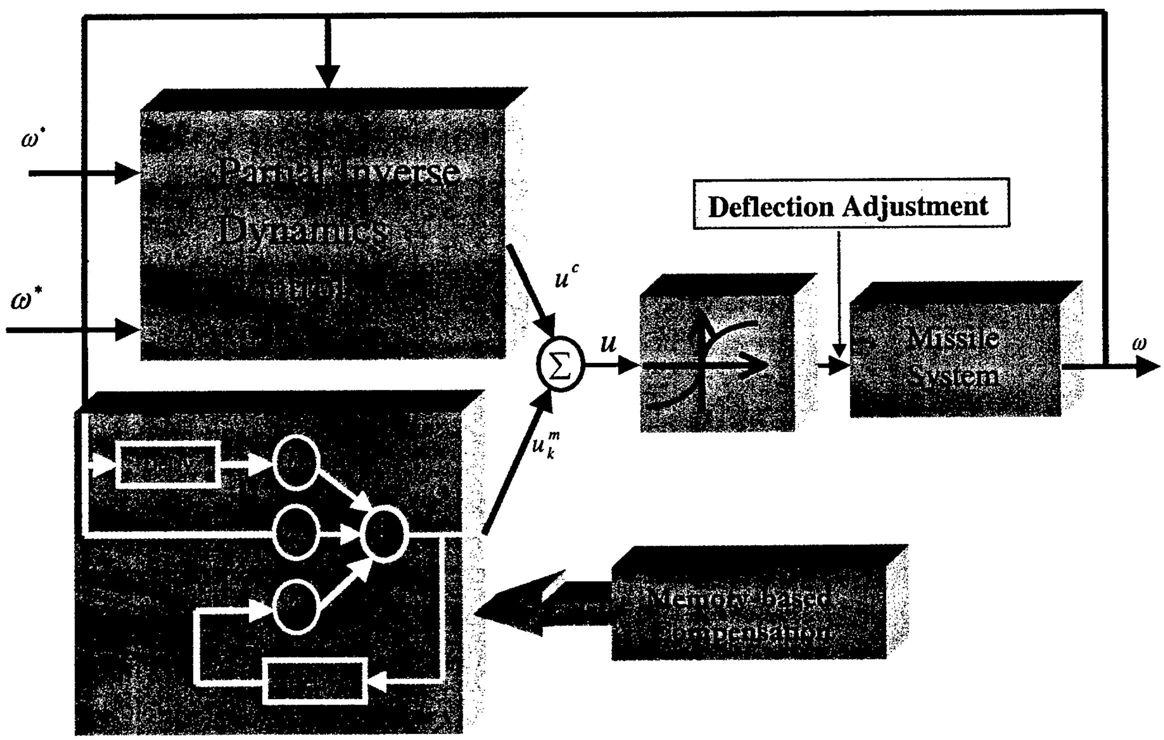

Figure 11 - Autopilot with memory-based compensation

The autopilot control block diagram with memory-based compensation is depicted in Figure 11. It is seen that only one-step back control history and system response are needed in the scheme. It is also stressed that the proposed memory-based compensation method does not involve ν_2 or ε_2 and no calculation/estimation of such parameters is needed. Only three control parameters k_0, k_s and k_z are to be selected.

6. Performance Evaluation

To verify the effectiveness of the proposed control algorithms, simulations under various flight conditions were conducted. This section presents some of the results. For simplicity, the pure orientation tracking is considered. Namely, the tracking performance of the developed control algorithms for the case of yaw, pitch, and roll motions is tested. Figure 12 is the control block diagram for this case. The nominal flight conditions simulated are: air density $\rho = 5.87e^{-04} slug / ft^3$, Altitude $=30{,}000 ft$, Mach number $M = 2.0$. The control parameters are $k_0 = 15$, $k_s = 15$, and $k_z = 15$. Since the flight conditions vary during the missile flight, the following parameters are used in the simulation model:

$$c = c_0 + \delta c$$

For instance, the air pressure is of the form

$$Q = Q_0 + \delta Q$$

where Q_0 is given as in [1], but δQ varies with time. To reflect the fact of varying flight conditions, the dynamic parameters $C_{m\delta_q}$ and $C_{l\beta}$, and the flight speed V as well as the air pressure Q as shown in Figures 13-16 were used for the test. The desired trajectory for the orientation is

$$\Theta^* = \begin{bmatrix} \phi^* \\ \theta^* \\ \psi^* \end{bmatrix} = \begin{bmatrix} 40(1-e^{-t}) \\ 30(1-e^{-t}) \\ 20(1-e^{-t}) \end{bmatrix} \quad \text{(degree)}$$

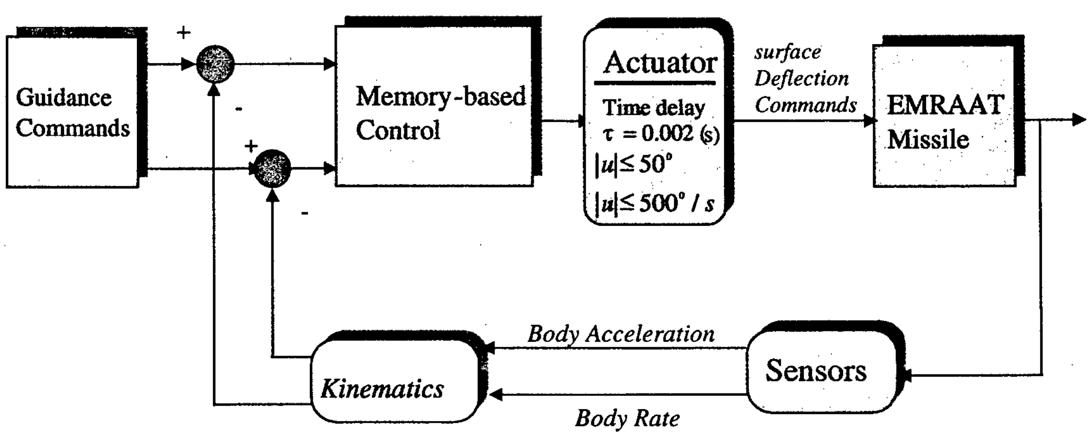

Figure 12 - Autopilot for orientation tracking

The simulation results are shown in Figures 17-21, where tracking performance is shown in Figures 17-19. It can be seen that the actual trajectory of missile tracks the guidance commands closely and timely. Figures 20-21 are the angle of attack and angle of the sideslip. All the control signals are indeed smooth and bounded.

7. Conclusion and Discussion

This project has investigated a memory-based approach for autopilot control of missile systems. The control scheme, purely built upon past control experience and most recent system responses, is quite effective in dealing with system nonlinearities, modeling uncertainties and external disturbances. The proposed method demonstrates robust and adaptive properties, while these properties are achieved through an avenue somewhat different from traditional methods -- there is no need for parameter estimation, repetitive actions, infinite switching frequencies, ad hoc design process or extensive off-line training. It is noted that the structure of the controller remains unchanged for different nonlinear functions $\tilde{\Delta}(.)$ (due to the change of flight conditions). Furthermore, the memory size does not grow with time, which could significantly facilitate real-time implementation. While memory-based approach for solving engineering problems has a long history, applying this method to control systems is a new attempt. Our current results show that memory-based control has potential for missile control applications. In this work, we investigated a method for autopilot design of missile system. It is shown that, with the proposed control algorithms, stable trajectory tracking is ensured. Unlike most existing work, where the actuator dynamics are ignored, we explicitly considered the effects of the actuator dynamics, which play an important role in system performance. The variation of aerodynamic coefficients due to varying flight conditions is considered in deriving the control algorithms. The effectiveness of the proposed control strategy has been verified via computer simulation.

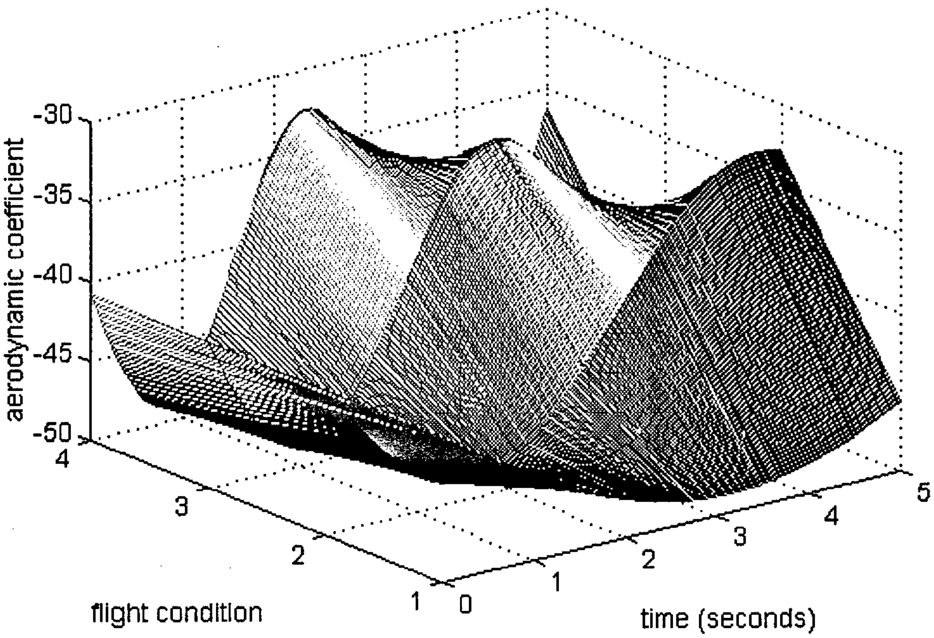

Figure 13 - The variation of $C_{m_{\delta_q}}$ with flight conditions

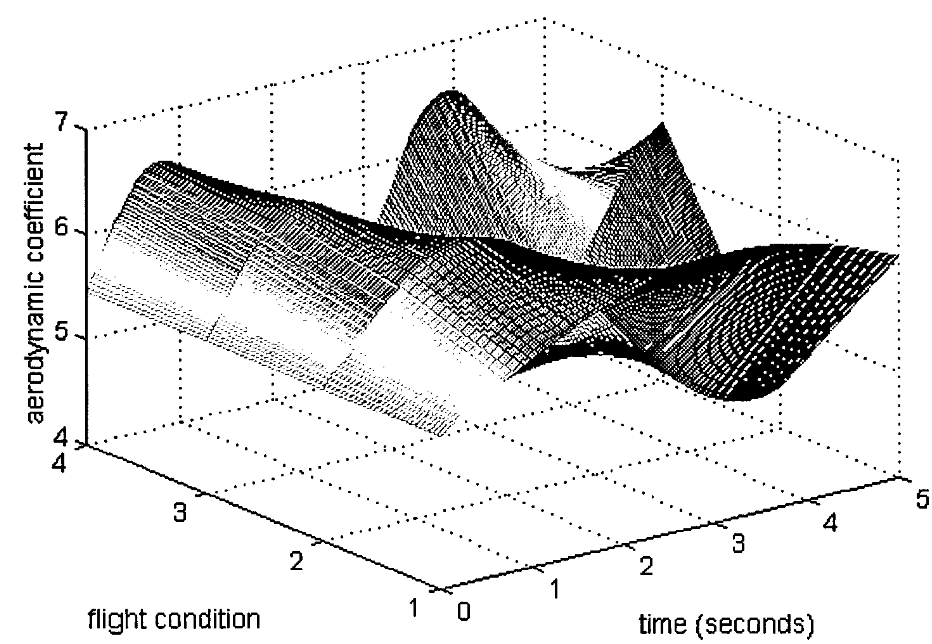

Figure 14 - The variation of $C_{l\beta}$ with flight conditions

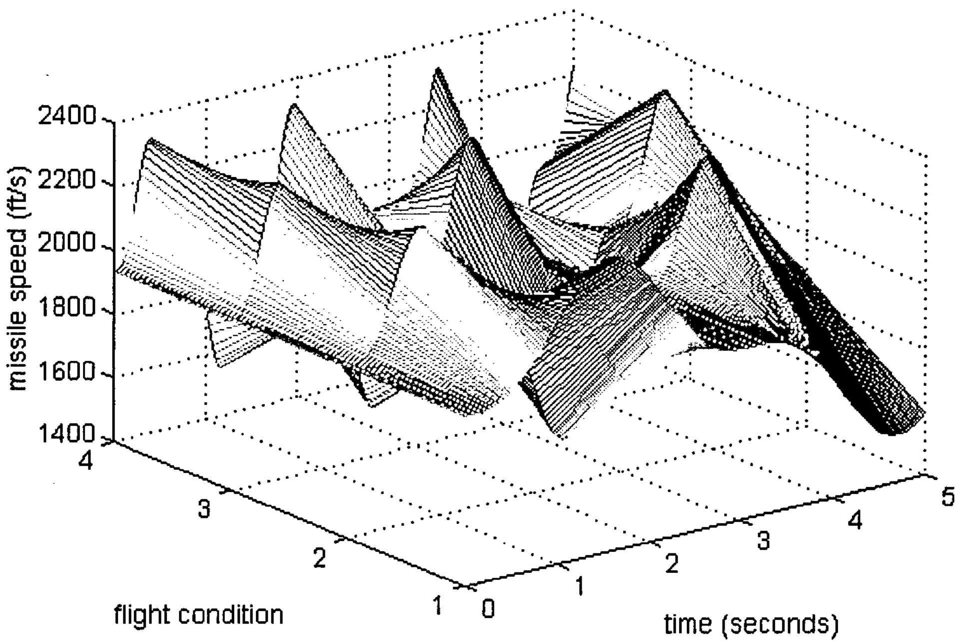

Figure 15 - The variation of missile speed V with flight conditions

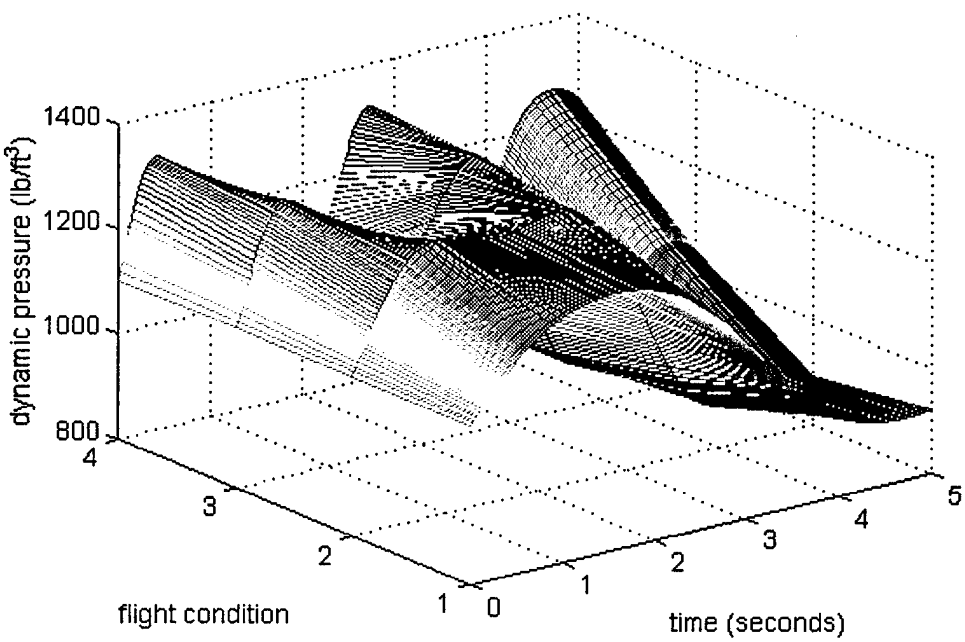

Figure 16 - The variation of air pressure Q with flight conditions

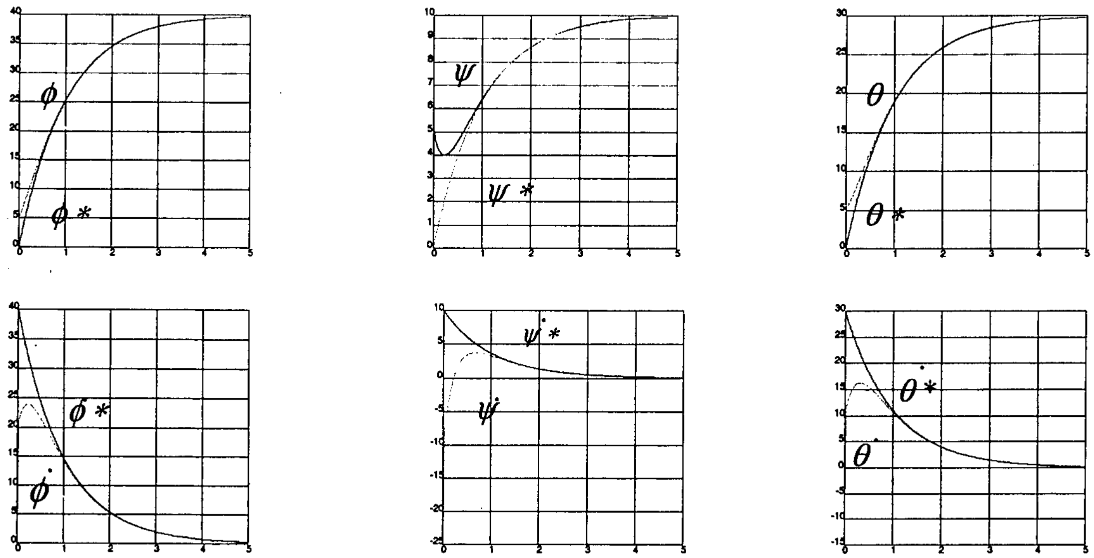

Figure 17- Yaw-pitch-roll motion tracking

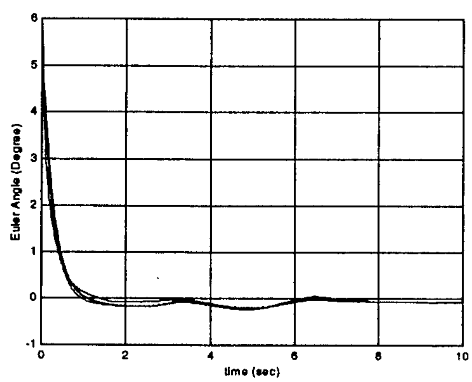

Figure 18 – Euler angle tracking error

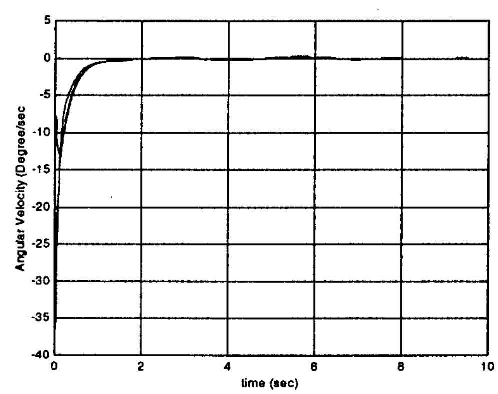

Figure 19 – Angular velocity tracking error

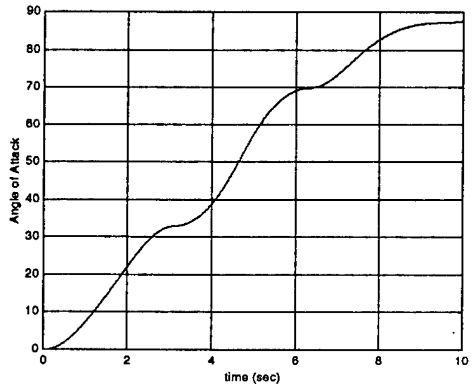

Figure 20 – Angle of attack vs. time

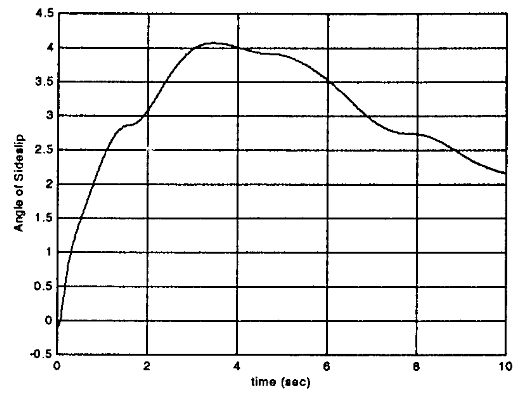

Figure 21 – Angle of sideslip vs. time

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Y. D. Song and J. Hou, "Autopilot Design Considering Actuator Dynamics in Missile Systems" **SPIE's 14th International Symposium on Aerospace/Defense Sensing, Simulation and Control**.

Y. D. Song, J. Hou and F. Ferguson, "Missile tracking with Memory based Compensation," **IEEE Transactions on Aerospace and Electronic Systems** (to be submitted).